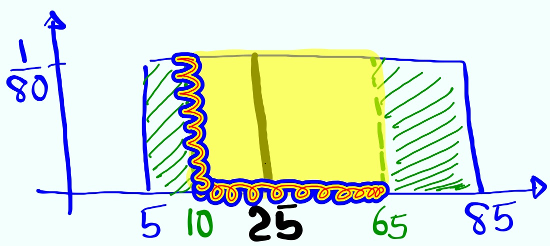
Statistics

Lecture 19

Feb 19-8:47 AM

Consider a uniform Prob. dist. for all values from 5 to 85.

1) Draw & clearly label.



2) $P(x=25)=0$

3) $P(x<10 \text{ or } x>65)$

$=1-P(10<x<65)$

$=1-(65-10)\cdot\frac{1}{80}$

$=1-\frac{55}{80}=\frac{25}{80}$

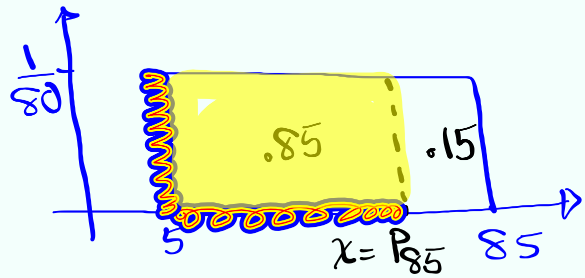
$=\boxed{\frac{5}{16}}$

Nov 4-12:13 PM

4) Find $x = P_{85}$

85% below — Left Area .85

15% above — Right Area .15



$(x-5)\cdot\frac{1}{80} = .85$

$x - 5 = 80(.85)$

$x = 5 + 80(.85)$

$\boxed{x = 73}$

Nov 4-12:19 PM

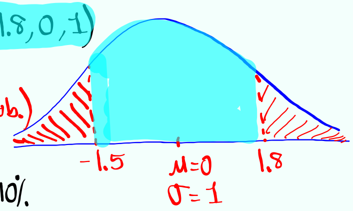
Find $P(\underbrace{Z < -1.5 \text{ and } Z > 1.8}) = 0$

Impossible event

Find $P(Z < -1.5$ OR $Z > 1.8)$

$= 1 -$ normalcdf(-1.5,1.8,0,1)

Total Area (Total prob.)



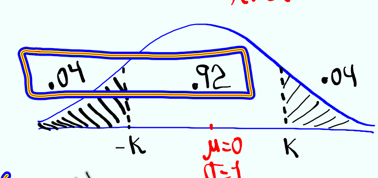
$= \boxed{.103} \approx 10\%$

Find K such that $P(\underbrace{-K < Z < K}) = .92$

Middle Area

$1 - .92 = .08$

$.08 \div 2 = .04$



K= invNorm(.96, 0, 1) = $\boxed{1.751}$

Nov 4-12:23 PM

Given $N(125, 15)$

Normal Prob. dist.

$\mu = 125$

$\sigma = 15$

$P(x > 110)$

$P(x < 140)$

$= \text{normalcdf}(110, E99, 125, 15)$

$= \boxed{.841}$

$\boxed{\text{2nd}}\boxed{,}$

$\text{normalcdf}(-E99, 140, 125, 15)$

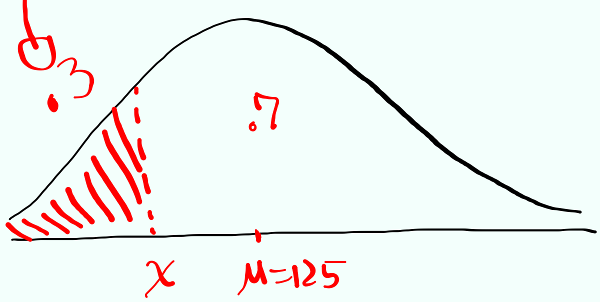
$(-)$

$= \boxed{.841}$

Nov 4-12:32 PM

Find x-value that separates the bottom 30% from the rest.

Round to whole #.



$x = \text{invNorm}(.3, 125, 15) \approx \boxed{117}$

left Area

μ

σ

Nov 4-12:39 PM

Credit Scores are normally distributed with mean of 710 and standard dev. of 60.

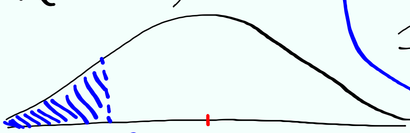
$N(710, 60)$

If one person is randomly selected, find the Prob. that his/her credit score is

x

a) below 600

$P(x<600)$

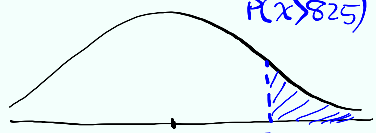


normalcdf(-E99,600,710,60)

= $\boxed{.033}$

b) above 825.

$P(x>825)$



normalcdf(825,E99,710,60)

= $\boxed{.028}$

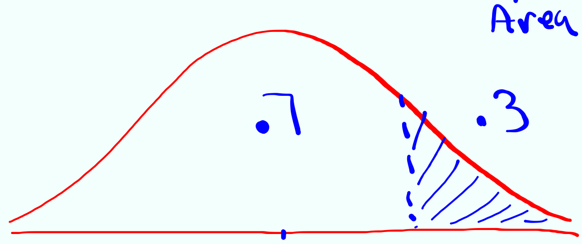
Nov 4-12:43 PM

Find a credit score that separates the top 30% from the rest.

x

Round to whole #.

Right Area



$x = \text{invNorm}(.7, 710, 60)$

$\approx \boxed{741}$

Nov 4-12:51 PM

Store
3, 5, 7, and 9
in L1, Use $\boxed{\text{1-Var Stats}}$
with L1 only to find

$\mu = \bar{x} = 6$

$\sigma = \sigma_x = 2.236$

$\sigma^2 = 5$

Take all Samples of Size 2
with replacement from this
list.

Find $\bar{x}$ of each Sample

3,3	3,5	3,7	3,9	3	4	5	6
5,3	5,5	5,7	5,9	4	5	6	7
7,3	7,5	7,7	7,9	5	6	7	8
9,3	9,5	9,7	9,9	6	7	8	9

Nov 4-12:55 PM

3	4	5	6
4	5	6	7
5	6	7	8
6	7	8	9

16 Means

$\bar{x}$	$P(\bar{x})$
3	1/16
4	2/16
5	3/16
6	4/16
7	3/16
8	2/16
9	1/16

$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$
with L2 & L3
Find

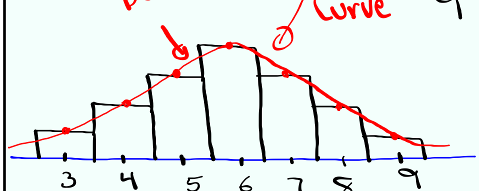
$\mu = 6$

$\sigma = 1.581$

$\sigma^2 = \frac{5}{2}$

Bell shape

Normal Curve



Central limit Theorem

$\boxed{\mu_{\bar{x}} = \mu}$ $\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$ $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

Nov 4-1:01 PM

Credit Scores had a normal dist

$\mu = 710$ & $\sigma = 60$

If we take random Samples of Size 4,

$\mu_{\bar{x}} = \mu = 710$

$\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{60}{\sqrt{4}} = \dfrac{60}{2} = 30$

Ages of teachers are N.D. with $\boxed{\mu = 52}$ and $\boxed{\sigma = 8.}$

$n = 16$

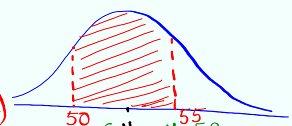
If we randomly select $\boxed{\text{16 teachers}}$ find the prob. that $\boxed{\text{their mean age}}$ is between 50 & 55.

$\bar{x}$

$P(50 < \bar{x} < 55)$

$= \text{normalcdf}(50, 55, 52, 2)$

$= \boxed{.775} \approx 78\%$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 52 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{16}} = \dfrac{8}{4} = 2 \end{cases}$

Nov 4-1:11 PM

Store 3, 5, 7, 9, and 11 in L1, use $\boxed{\text{1-Var Stats}}$ with L1 only to find

$\mu = 7$

$\sigma = 2.828$

$\sigma^2 = 8$

Take all Samples of Size 2 with replacement.

3,3	3,5	3,7	3,9	3,11
5,3	5,5	5,7	5,9	5,11
7,3	7,5	7,7	7,9	7,11
9,3	9,5	9,7	9,9	9,11
11,3	11,5	11,7	11,9	11,11

Now find $\bar{x}$ of each Sample

3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10
7	8	9	10	11

Nov 4-1:32 PM

3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10
7	8	9	10	11

25 Means

$\bar{x}$	$P(\bar{x})$
3	1/25
4	2/25
5	3/25
6	4/25
7	5/25
8	4/25
9	3/25
10	2/25
11	1/25

$\bar{x} \to L2$

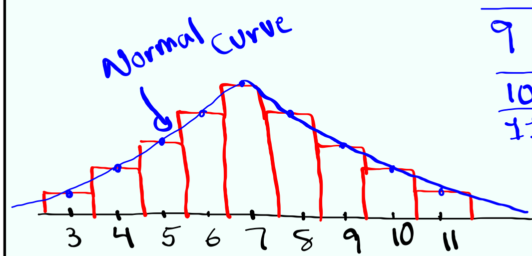
$P(\bar{x}) \to L3$

Use $\boxed{\text{1-Var Stats}}$ with L2 & L3 to find

$\mu = 7$

$\sigma = 2$

$\sigma^2 = 4 = \frac{8}{2}$



CLT (Central Limit Theorem)

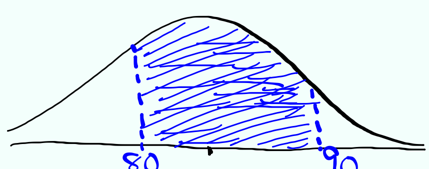
$\boxed{\mu_{\bar{x}} = \mu}$ $\boxed{\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}}$ $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

Nov 4-1:38 PM

Scores on a math exam were normally dist. with mean of 84 and standard dev. of 6. $N(84, 6)$

If we randomly select $\overset{n=5}{\boxed{\text{5 exams}}}$ find the Prob. that $\overset{\bar{x}}{\boxed{\text{their mean score}}}$ is between 80 and 90.

$P(80 < \bar{x} < 90)$



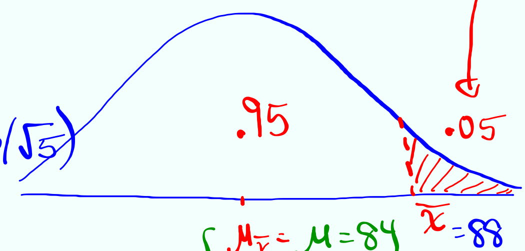
$=$ normalcdf$(80, 90, 84, 6/\sqrt{5})$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 84 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{5}} \end{cases}$

$= \boxed{.919} \approx 92\%$

Nov 4-1:46 PM

For randomly selected groups of 5 exams,
find the mean score that separates the top 5%
from the rest.



$\bar{x} = \text{invNorm}(.95, 84, 6/\sqrt{5})$

$\approx \boxed{88}$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 84 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{5}} \end{cases}$

SG 18, 19, 20, 21.

Nov 4-1:53 PM

$Z_{\alpha/2}$ — $\alpha/2$ is the area on the right tail of normal curve with $\mu = 0, \sigma = 1$

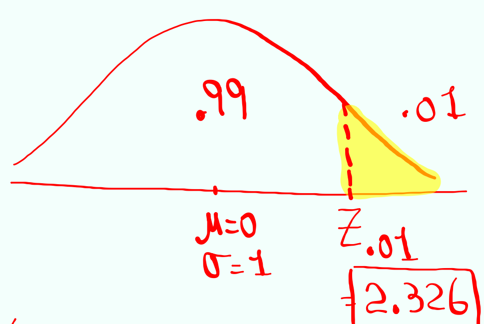
Alpha $0 < \alpha < 1$

Significance level

Find $Z_{.01}$

$\frac{\alpha}{2} = .01$

$\alpha = .02$



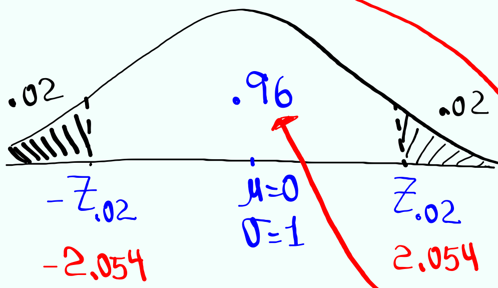
$Z_{.01} = \text{invNorm}(.99, 0, 1)$

Nov 4-1:59 PM

Find $\pm Z_{.02}$

$Z_{.02} = \text{invNorm}(.98, 0, 1)$

$= \boxed{2.054}$



$\frac{\alpha}{2} = .02$

$\alpha = .04$

$1-\alpha = .96$

$\alpha \rightarrow$ Significance level

$\alpha/2 \rightarrow$ Area of each tail

$1-\alpha \rightarrow$ Middle area

$(1-\alpha)\cdot 100\% \rightarrow$ Confidence level

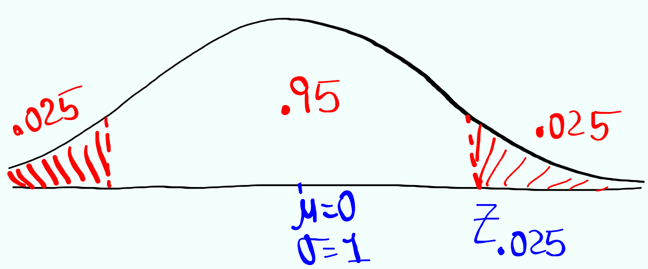
Nov 4-2:03 PM

Suppose $\alpha = .05$ Significance level

$\alpha/2 = .025$ area of each tail

$1-\alpha = .95$ Middle area

$(1-\alpha)\cdot 100\% = 95\%$ Confidence level

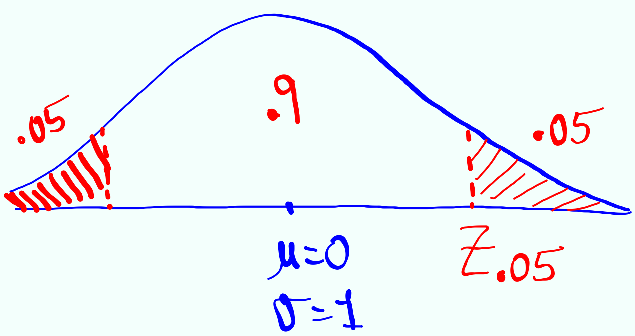


$Z_{.025} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Nov 4-2:09 PM

Find $z_{\alpha/2}$ for $\underbrace{\text{90\% Confidence level.}}_{\text{Middle Area}}$

.9



$1-\alpha=.9$

$\alpha=.1$
Sig. level

$\alpha/2=.05$

$=\text{invNorm}(.95,0,1)$

$=\boxed{1.645}$

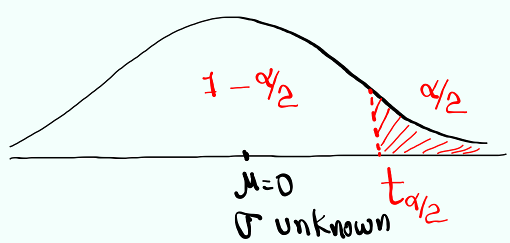
Nov 4-2:13 PM

t - Dist

1) Symmetric, Bell-Shape, Total area = 1

2) $\mu=0$, σ is unknown

3) It comes with degrees of freedom.



df

We use

invT(Left Area, df)

$\boxed{\text{2nd}}\ \boxed{\text{VARS}}$

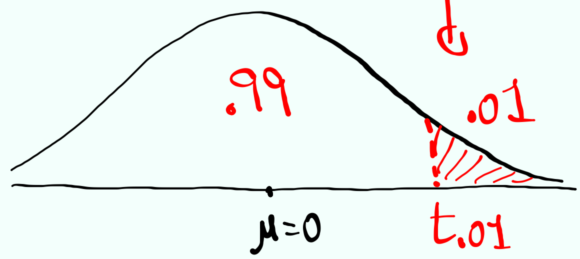
Nov 4-2:16 PM

Find $t_{.01}$ with $df = 9$

$invT(.99, 9)$

Left Area ↑ ↑ df

$\approx \boxed{2.821}$



σ unknown
df=9

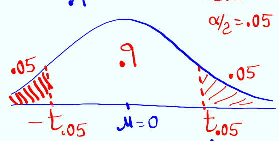
Nov 4-2:21 PM

Find $\pm t_{\alpha/2}$ for 90% C-level with $df=15$.

Middle Area
.9

$1-\alpha = .9$
$\alpha = .1$
$\alpha/2 = .05$

$invT(.95, 15)$
$= \boxed{1.753}$



σ unknown
df=15

What is degrees of freedom?

23 students

I bring 23 donuts

$\boxed{df = 22}$

First	student	23	choices
Second	"	22	"
⋮	⋮	⋮	⋮
Last	"	1	donut left (0 choices)

Luke has 7 clean T-Shirt.

Monday → 7 choices

Tuesday → 6 choice

⋮

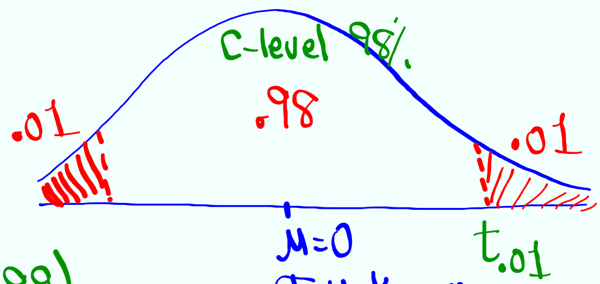
Sunday → 0 choices (1 clean T-Shirt)

$\boxed{df = 6}$

Nov 4-2:26 PM

Find $t_{\alpha/2}$ for $\alpha = .02$ with $df = 99$.

$\alpha/2 = .01$



$t_{.01} = \text{invT}(.99, 99)$

$= \boxed{2.365}$

σ unknown

$df = 99$

As df increase, $\Rightarrow t_{\alpha/2} \to z_{\alpha/2}$

Nov 4-2:35 PM